

June 2023

Dear Future Geometry Students and Parents:

Welcome to Geometry! For the 2023-2024 school year, we would like to focus your attention to the fact that many concepts from Algebra I are infused into Geometry. In order to be successful in Geometry, a student must demonstrate a proficiency in certain skills including:

- Multiplying Polynomials
- Factoring/ Solving Quadratics
- Solving Systems of Equations
- Determining the Equations of Lines
 - ✓ Parallel Lines
 - ✓ Perpendicular Lines
- Simplifying Radicals

These topics will be reviewed briefly throughout the coming year; however, the topics are not re-taught in the Geometry course. To ensure that all students demonstrate the basic algebra skills to be successful, an assessment will be administered within the first **two weeks** of school (date to be determined). The attached review packet is provided for practice and is intended as a tool for assessment readiness. In addition, students are encouraged to seek extra help before or after school from their teacher for any topics requiring more personal in-depth remediation. Additional practice problems and explanations may also be found beginning on page 869 of the Geometry textbook.

It is expected that each student will fully complete the review packet. Packets will be collected in class on the day of the assessment. If you have any questions, please do not hesitate to contact your child's teacher.

Mrs. Fauerbach Mrs. Osmer

Binomial Products

To multiply two binomials, you can use the Distributive Property systematically. You may know this as the "**FOIL**" method or "**Area Model**".

Example: Find the product

Simplify (x + 2)(3x - 4) = x(3x) + x(-4) + 2(3x) + 2(-4)Distribute $= 3x^{2} - 4x + 6x - 8$ Simplify--multiply $= 3x^{2} + 2x - 8$ Simplify--combine like terms

Simplify $(7a - 1)^2$

= (7a - 1)(7a - 1)Rewrite as the product of the binomial and itself= 7a(7a) + 7a(-1) + (-1)(7a) + (-1)(-1)Distribute= $49a^2 - 7a - 7a + 1$ Simplify--multiply= $49a^2 - 14a + 1$ Simplify--combine like terms

<u>Simplify</u>

1) $(x-1)^2$

2) $(3x + 4)^2$

3) $(-9a^2 - 2x)^2$

Factoring

Factoring (Common Monomial Factor)

Solution:

- The first step in ANY factoring problem is to factor out the GCF. The other factor is the original polynomial divided by the GCF.
- A variable must be in each term of the polynomial in order to be in the GCF. Always use the smallest exponent in the GCF.
- If there is no common factor, the polynomial can be "**prime**."

Example: Factor completely $24xy + 18xy^2 - 3y$

$24xy + 18xy^2 - 3y$	Initial polynomial
GCF = 3y	Find the GCF
$24xy + 18xy^2 - 3y$	Determine other factor by
$\frac{3}{3v} = 8x + 6xy - 1$	dividing each term by the
	GCF
3y(8x+6xy-1)	Final factorization
	Look to see if the polynomial
	factors further.

Factor Completely

- 4) $3y^2 9y$
- 5) $54x^5y^3z^2 9x^2y^6r^3 + 36x^4r^7$

6)
$$5x^2y^2 - 9r^3 + 7z$$

Factoring (trinomial where a = 1)

Solution:

• Put expression into standard form, if not already done.

Standard form: $ax^2 + bx + c$

- If necessary, factor out the GCF or a "-1" (if the *a* is negative). Then look to see if you can factor further.
- Look to see if the resulting trinomial can be factored further. For any trinomial of form x² + bx + c, it will factor if there are two factors of c that add up to b (i.e., factors f₁ and f₂, such that f₁ · f₂ = c and f₁ + f₂ = b). The trinomial can then be factored as (x+f₁)(x+f₂)
- When finding the factors f_1 and f_2 , remember that if c is negative, f_1 and f_2 must have different signs (one positive and one negative). If c is positive, then f_1 and f_2 must both have the same sign as b.
- If there are <u>no two such factors</u>, then the trinomial is "**prime**."

$2x^2 - 20x + 42$	Standard Form, if necessary.
$2(x^2-10x+21)$	Factor out the GCF of 2.
$f_1 = -7, \ f_2 = -3$	Find the factors of +21 that add up to -10. They must both be negative.
2(x-7)(x-3)	 Use f₁ and f₂ to factor the trinomial. *As part of the final answer, <u>don't</u> forget to bring down the G.C.F.*

Example: Factor completely $2x^2 - 20x + 42$

7) $x^2 - 5x + 6$	8) $x^2 + 23x + 42$

9) $x^2 - 10x + 24$

10) $-7x + x^2 + 20$

11) $3x^2 + 15x + 18$

12) $-x^2 - 5x + 50$

Solution:

There are several methods for factoring these types of trinomials. If you prefer another method, see your teacher if you have any questions.

Example: Factor completely $-5x-6+6x^2$

Method 1: Guess & Check

Method 2: AC Method

$-5x-6+6x^2$	Initial trinomial
$6x^2 - 5x - 6$	Step 1: Put into standard form, if necessary
not necessary	Step 2: Factor out GCF or -1, if necessary
(6)(-6) = -36	Step 3: Multiply <i>a</i> · <i>c</i>
$f_1 = 4, f_2 = -9$	Step 4: Find the factors of the product of $a \cdot c$ that add up to b (i.e., factors f_1 and f_2 , such that $f_1 \cdot f_2 = a \cdot c$ and $f_1 + f_2 = b$). Remember that if $a \cdot c$ is negative, f_1 and f_2 must have different signs (one positive and one negative). If $a \cdot c$ is positive, then f_1 and
	f_2 must both have the same sign as b .
$6x^2 - 5x - 6$ = $6x^2 + 4x - 9x - 6$	Step 5: Rewrite the middle term as the sum of two terms whose coefficients are the numbers above.
= 2x(3x+2) - 3(3x+2) * * = (3x+2)(2x-3)	<pre>Step 6: Factor by grouping **the expression in the parentheses must be the same</pre>

Factor completely using the method of your choice.

13) $2x^2 - 11x + 5$

14) $8x^2 + 6x - 9$

15) $-8x^2 + 6x + 2$

Factoring (binomial)

Solution: $a^2 - b^2 = (a+b)(a-b)$

"Difference of Two Perfect Squares"

- > Must be a binomial (two terms)
- > Terms must be subtracted
- > Each term must be a perfect square

A binomial with the <u>sum</u> of two perfect squares is **prime.

Example: Factor completely $81x^2 - 16$

$81x^2 - 16$	Original binomial	
not necessary	Factor out G.C.F. or -1, if necessary	
$81x^2 - 16$	Are the terms subtracted? YES!	
$81x^2 = (9x)^2$ $16 = (4)^2$	Are the first and last terms perfect squares? YES!	
$81x^2 - 16 = (9x + 4)(9x - 4)$	Answer is the product of the sum and difference of the square roots.	

Factor completely

16) $25x^2 - 1$

17) $100-36y^2$

18) $64y^4 + 49x^6$

Solution:

- Re-write the quadratic in standard form $ax^2 + bx + c = 0$.
- Factor the polynomial completely
- Set each factor equal to zero (use "or" between the equations)
- Solve each resulting equation to obtain *two* solutions.

Example: Solve $6x^2 - 5x = 6$

$6x^2 - 5x = 6$	Original equation
$6x^2 - 5x - 6 = 0$	Write in standard form
(2x-3)(3x+2) = 0	Factor the polynomial completely
2x - 3 = 0 or $3x + 2 = 0$	Set each factor equal to zero
$x = \frac{3}{2}$ or $x = -\frac{2}{3}$	Solve both equations

Tip: If you have a negative sign in front of the x^2 term, simply multiply both sides of the equation by -1, then go ahead and factor. Similarly, divide both sides by any GCF. For example, change $-18x^2+15x+18=0$ to $6x^2-5x-6=0$ by dividing both sides by -3.

Solve

19) $-16 = x^2 + 10x$

20)
$$-2x^2 + 70 = -4x$$

21) $24x^2 + 10x - 6 = 0$

Solve using the Quadratic Formula.

Quadratic Formula:
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve using the Quadratic Formula: $15x + 3x^2 + 18$

$3x^{2} + 15x + 18 = 0$ a = 3, b = 15, c =18	← set equal to zero, and in standard form, if not already done.← Identify a, b and c.
$x = \frac{-15 \pm \sqrt{15^2 - 4 \cdot 3 \cdot 18}}{2 \cdot 3}$	\leftarrow Substitute into the quadratic formula
$x = \frac{-15 \pm \sqrt{225 - 216}}{6}$	\leftarrow Simplify (remember the order of operations!)
$x = \frac{-15 \pm \sqrt{9}}{6}$	
$x = \frac{-15 \pm 3}{6}$	
$x = \frac{-15+3}{6}$ or $x =$	$\frac{-15-3}{6} \leftarrow \text{Split up the } \pm$
$x = \frac{-12}{6} \qquad \text{or} \qquad x = -\frac{1}{6}$	$\frac{-18}{6} \leftarrow \text{Simplify each answer}$
$\mathbf{x} = -2 \qquad \text{or} \qquad \mathbf{x} = -3$	

<u>Solve</u>

22) $3x^2 - 5x - 6 = 0$ 23) $-7x + x^2 = 20$

Graphing

Determine and graph the equation of a line y = mx + b

Solution:

- Determine slope from the two given points using the formula $m = \frac{y_2 y_1}{x_2 x_1}$.
- Determine the *y*-intercept (b) by using y = mx + b
 - \checkmark substitute a point for the *x* and *y* variables
 - \checkmark substitute the slope for the *m* variable
 - ✓ solve for b
- Rewrite the equation y = mx + b by replacing the *m* and *b* variables with the values you found.
- Use the slope and *y*-intercept to graph.
- Example: Determine and graph the equation of the line passing through the points (2, 7) and (-1, -2).

$m = \frac{7 - (-2)}{2 - (-1)} = \frac{9}{3} = 3$ $m = \frac{3}{1} \text{ or } \frac{-3}{-1}$	Determine slope
y = mx + b $7 = (3)(2) + b$ $7 = 6 + b$ $1 = b$	Determine the <i>y</i> -intercept —you may use either point
y = 3x + 1	Write the equation
Graph on the next page	Graph equation using <i>y</i> -intercept and slope



Determine and graph the equation of the line passing through the given point





25) (3, -1) and (6, 7)



26) (-2, -6) and (-2, 4)



Determine the equation of a line parallel or perpendicular to another line.

Example: Determine the equation of a line in slope-intercept form which is perpendicular/parallel to the line 2x - 3y = 8 and passes through the point (-1, 5).

Solution:

Step 1: Determine the slope of the given equation by writing in slope-intercept form. 2x-3y=8



Step 2: Determine the slope of the perpendicular line and parallel line



Step 3: Using the slope from step 2 and the given point, determine the y-intercept (b).

Parallel

Parallel

Perpendicular

y = mx + b $5 = \left(-\frac{3}{2}\right)(-1) + b$ $5 = \frac{3}{2} + b$ $\frac{7}{2} = b$ y = mx + b $5 = \left(\frac{2}{3}\right)(-1) + b$ $5 = -\frac{2}{3} + b$ $\frac{17}{3} = b$

Step 4: Write the equation of the line by substituting your values for *m* and *b*.

Perpendicular

$$y = -\frac{3}{2}x + \frac{7}{2}$$
 $y = \frac{2}{3}x + \frac{17}{3}$

27) Determine the equation of a line in slope-intercept form which is perpendicular to the line x+4y=8 and passes through the point (2, -6).

28) Determine the equation of a line in slope-intercept form which is parallel to the line -4x-3y = -1 and passes through the point (-3, -3).

29) Determine the equation of a line in slope-intercept form which is perpendicular to the line -4x+2y=8 and passes through the point (1, 6).

System of Equations

Solution:

There are three ways to solve a system of equations:

- 1. Graphing Method
- 2. Substitution Method
- 3. Elimination Method

We will demonstrate the substitution and elimination methods.

Example: Consider the system of equations: $\begin{cases} -2x + y = 4 \\ x + y = -5 \end{cases}$

Substitution method:

- Step 1: Solve for **one** variable in **one** of the equations.
- Step 2: Substitute the expression into the **OTHER** equation.
- Step 3: Solve your new equation.
- Step 4: Substitute the value of variable into one of the original equations to determine second variable.
- Step 5: Write your answer as an ordered pair.





Elimination Method:

- Step 1: Line up the variables on one side and the constants on the other side.
- Step 2: Determine a variable you want to eliminate—you want the coefficients to be the same number with opposite signs.
- Step 2: If necessary, multiply one or both equations by a number to force the coefficients to be the same number with opposite signs.
- Step 3: **<u>Add</u>** the equations together so that one variable cancels out.
- Step 4: Solve the equation.
- Step 5: Substitute the value of variable into one of the original equations to determine the value of the second variable
- Step 5: Write your answer as an ordered pair.



Solution: (-3, -2)

Solve the following system of equations by substitution.

30)
$$\begin{cases} x + 2y = 12 \\ x - y = -3 \end{cases}$$
 31)
$$\begin{cases} -4x + 3y = -19 \\ 2x + y = 7 \end{cases}$$

Solve the following system of equations by elimination.

32) $\begin{cases} 2x - 3 \\ 6x - 9 \end{cases}$	y = 8 y = 12	33)	$\begin{cases} 8x - 5y = 14 \\ 10x - 2y = 9 \end{cases}$
(6x - 9)	$r \equiv 12$,	(10x - 2y = 9)

	Radicals	
Simplify Radicals	Radic)
	index radicand	/
	VIUULUIIU	

Solutions:

- You will need to know all your perfect squares $1^2 = 1$ through $20^2 = 400$.
- To simplify $\sqrt{radicand}$
 - = √perfect square non-perfect square
 = √perfect square √non-perfect square
 = square root of perfect square √non-perfect square

Examples: Simplify the following

1.
$$\sqrt{121}$$

= 11
= $4\sqrt{3}$
2) $\sqrt{48}$
= $\sqrt{16 \cdot 3}$
= non-real answer

4)
$$-5\sqrt{128}$$
$$= -5\sqrt{64 \cdot 2}$$
$$= -5 \cdot \sqrt{64} \cdot \sqrt{2}$$
$$= -5 \cdot 8 \cdot \sqrt{2}$$
$$= -40\sqrt{2}$$

Add/Subtract Radicals

Solution:

- In order to add or subtract radicals, the index and the radicand must be the same!
- You combine radicals as if you were combining like-terms—add/subtract their coefficients **ONLY**.
- Sometimes it may be necessary to simplify the radicals before you add or subtract.

Examples: Simplify. Express your answer in simplified radical form.

- 1) $5\sqrt{13} 8\sqrt{13}$ $= -3\sqrt{13}$ $= -\sqrt{9 \cdot 11} + 3\sqrt{11}$ $= -\sqrt{9 \cdot \sqrt{11}} + 3\sqrt{11}$ $= -\sqrt{9 \cdot \sqrt{11}} + 3\sqrt{11}$ $= -3\sqrt{11} + 3\sqrt{11}$ = 0
- 3) $\sqrt{175} 2\sqrt{28}$ $= \sqrt{25 \cdot 7} - 2\sqrt{4 \cdot 7}$ $= \sqrt{25} \cdot \sqrt{7} - 2 \cdot \sqrt{4} \cdot \sqrt{7}$ $= 5\sqrt{7} - 2 \cdot 2\sqrt{7}$ $= 5\sqrt{7} - 4\sqrt{7}$ $= \sqrt{7}$

Multiplication/Division of Radicals

Solution: **This is NOT the only way to simplify these problems**

- Simplify the radical first.
- For multiplication:
 - ✓ Multiply numbers outside the radical together
 - \checkmark Multiply numbers under the radical together
 - \checkmark Make sure to check if your radical needs to be simplified again
- For division:
 - ✓ Divide numbers outside the radical
 - \checkmark Divide numbers under the radical

О

 $\checkmark~$ Make sure to check if your radical needs to be simplified again

Examples: Simplify. Express your answer in simplified radical form.

1)
$$\frac{\sqrt{192}}{\sqrt{3}} = \frac{\sqrt{64 \cdot 3}}{\sqrt{3}} = \frac{\sqrt{64} \cdot \sqrt{3}}{\sqrt{3}} = \frac{8\sqrt{3}}{\sqrt{3}} = 8$$

2) $\sqrt{\frac{2}{49}} = \frac{\sqrt{2}}{7}$

3)
$$2\sqrt{6} \cdot -\sqrt{27}$$
$$= 2\sqrt{6} \cdot -\sqrt{9 \cdot 3}$$
$$= 2\sqrt{6} \cdot -\sqrt{9} \cdot \sqrt{3}$$
$$= 2\sqrt{6} \cdot -3\sqrt{3}$$
$$= (2) \cdot (-3)(\sqrt{6})(\sqrt{3})$$
$$= -6\sqrt{18} \circ \qquad \bigcirc$$
$$= -6\sqrt{9 \cdot 2}$$
$$= -6\sqrt{9} \cdot \sqrt{2}$$
$$= -6 \cdot 3 \cdot \sqrt{2}$$
$$= -18\sqrt{2}$$



34) $-\sqrt{169}$ 35) $\sqrt{80} - 14\sqrt{5}$

36)
$$-3\sqrt{2} \cdot \sqrt{50}$$
 37) $\frac{\sqrt{120}}{\sqrt{8}}$

38)
$$\sqrt{\frac{72}{9}}$$
 39) $5\sqrt{2} + 2\sqrt{128}$

40)
$$\frac{12}{\sqrt{3}}$$
 41) $\sqrt{\frac{2}{5}}$

Geometric Notation and Defintions

The followin	<u>The following set of notation and definitions will be used throughout the entire course.</u>			
Notation	Meaning	Diagram		
\overrightarrow{AB} or \overrightarrow{BA}	Line <i>AB</i> or Line ℓ	l l		
or Line ℓ	Has one dimension. Through any two points, there is exactly one line.	A B		
\overline{AB}	Segment AB			
$\frac{\text{or}}{BA}$	Consists of two endpoints A and B and all of the points on \overleftrightarrow{AB} between A and B	A B		
\overrightarrow{AB}	Ray AB			
Can't switch	Consists of one endpoint A and all the points	A B		
order!!	on AB that are on the same side as B			
AB or BA	The length of segment <i>AB</i> (has no segment bar on top)	AB = 5 m.		
=	Equal to	C 2 in. D E 2 in. F		
	*Lengths and angle measures are equal	EqualCongruent $CD = EF$ $\overline{CD} \cong \overline{EF}$		
≅	Congruent (has the same measure)	$B = 22^{\circ}$		
	*Segments and angles are congruent	ACongruentEqual $\measuredangle A \cong \measuredangle B$		
$\angle ABC$ or $\angle ABC$	Angle <i>ABC</i> has a vertex of B The vertex should be the middle letter			
m₄ABC or m∠ABC	The measure of angle <i>ABC</i>	$B \stackrel{A}{=} 40^{\circ}$ $C \qquad \qquad$		
o	Degree(s), a unit measure for angles	100°		
L	Perpendicular	É		
	Two lines that intersect to form a right angle.			
II	Parallel			
	Two <mark>coplanar</mark> lines that never intersect. They have the same slope.	$ \longleftrightarrow $		
ΔABC ΔCBA	Triangle <i>ABC</i>	$A \xrightarrow{B} C$		

Other Definitions

Point	A point has no dimension.	
Point A	It is represented by a dot	\bullet_A
A		
Plane	A plane has two dimensions. It is represented by a shape that looks like	• _A M
Plane ABC	a floor or a wall and extends without end.	
Plane M		В
	Through any three points not on the same line, there is exactly one plane.	
	You can use three points, not all on the same line, to name it.	Plane <i>ABC</i> or Plane <i>M</i>
	Sometimes, you can use a capital letter without a point, if it is provided.	
Opposite	Two rays with a common end point	
Rays	that go in opposite directions.	$\begin{array}{c c} & \bullet & \bullet \\ A & B & C \end{array}$
	The first letter is the endpoint.	\overrightarrow{BA} and \overrightarrow{BC} are opposite rays
Collinear Points	Points that are on the same line.	See diagram above for OPPOSITE RAYS.
		A, B, C are collinear points.
Coplanar Points	Points that are on the same plane.	See diagram for PLANE above.
		A, B, C are coplanar points
Angles	<u>Classifying angles:</u>	<u>Naming angles:</u>
	hetween 0° and 00°	1. Use 5 letters (middle letter is
	9 Bight: an angle with measure	2 Use the vertex letter (only
	2. Argint. an angle with measure equal to 90°	2. Use the vertex letter (only when one angle is present)
	3 Obtuse: an angle with	3 Sometimes an angle can be
	measure between 90° and 180°	named with a number
	4. Straight: an angle with	
	measure equal to 180°	A
		B



- 5. Acute: an angle with measure between 0° and 90°
- 6. Right: an angle with measure equal to 90°
- 7. Obtuse: an angle with measure between 90° and 180°
- 8. Straight: an angle with measure equal to 180°

Naming angles:

- 4. Use 3 letters (middle letter is always the vertex)
- 5. Use the vertex letter (only when one angle is present)
- 6. Sometimes an angle can be named with a number



Area and Perimeter

Terms to know:

- Area: how much is contained within a 2-dimensional, contained shape
- *Perimeter*: the distance around a 2-dimensional, contained shape
 - $\circ~$ for a circle, called circumference.
- Base and Height: one must be a side length. they must be perpendicular to each other.
- Area of a square (b·h), rectangle (b·h), triangle ($\frac{1}{2}$ b·h), circle (π r²)
- Perimeter of polygons (sum of all the sides)
- Circumference of a circle $(2\pi r)$
- Pythagorean Theorem: the relationship between the sides of a right triangle : $a^2 + b^2 = c^2$, where c is the hypotenuse



- 1) $x^2 2x + 1$
- 2) $9x^2 + 24x + 16$
- 3) $81a^4 + 36a^2x + 4x^2$
- 4) 3y(y-3)
- 5) $9x^2(6x^3y^3z^2 y^6r^3 + 4x^2r^7)$
- 6) prime
- 7) (x-2)(x-3)
- 8) (x+2)(x+21)
- 9) (x-4)(x-6)
- 10) prime
- 11) 3(x+3)(x+2)
- 12) -(x+10)(x-5)
- 13) (x-5)(2x-1)
- 14) (2x + 3)(4x 3)
- 15) -2(x-1)(4x+1)
- 16) (5x+1)(5x-1)
- 17) 4(5+3y)(5-3y) or -4(3y-5)(3y+5)
- 18) prime
- 19) x = -8 or x = -2
- 20) x = 7 or x = -5

21)
$$x = -\frac{3}{4}$$
 or $x = \frac{1}{3}$

- 22) $\frac{5\pm\sqrt{97}}{6}$
- 23) $\frac{7\pm\sqrt{129}}{2}$

Geometry Honors Summer Assignment



27)	y = 4x - 14
28)	$y = -\frac{4}{3}x - 7$
29)	$y = -\frac{1}{2}x + \frac{13}{2}$
30)	(2,5)
31)	(4, -1)
32)	Ø or No Solution
33)	$(\frac{1}{2}, -2)$
34)	-13
35)	$-10\sqrt{5}$
36)	-30
37)	$\sqrt{15}$
38) 2	$2\sqrt{2}$
39)	$21\sqrt{2}$
40)	$4\sqrt{3}$
41)	$\frac{\sqrt{10}}{5}$
42) /	$A = 64 in^2$ $P = 32 in$
43) /	A = $368 m^2$ P = $78 m$
44) .	$A = 54 \ cm^2$ $P = 36 \ cm$
45) /	A = $81\pi mi^2$ C = $18\pi mi$